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1994 J. Phys. A: Math. Gen. 27 L231

(http://iopscience.iop.org/0305-4470/27/8/002)

J. Phys. A: Math. Gen. 27 (1994) L231–L236. Printed in the UK

LETTER TO THE EDITOR

The restricted CPT gauge symmetry and the Weinberg–Salam model*

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Received 2 November 1993

Abstract. By means of an approach to the generalized gauge theory on discrete groups we take the CPT transformations as a gauge group with a CPT transformation as a single element and reformulate the Weinberg–Salam model with the Higgs being a gauge field with respect to this restricted CPT gauged symmetry.

Recently, we have generalized [1–3] the ordinary Yang–Mills gauge theory in order to take not only Lie groups but also discrete groups as gauge groups [4] and we have completed an approach to this generalized gauge theory in the spirit of, but different from, the non-commutative geometry approach [5–7]. We have shown that Higgs fields are such gauge fields with respect to discrete gauge symmetry over four-dimensional space-time, and the Yukawa couplings between Higgs and fermions may be introduced automatically via covariant derivatives with respect to discrete gauge potentials. We have also studied the Weinberg–Salam model for the electroweak interaction and the standard model for the electroweak–strong interaction [2, 3]. In all these models, Higgs appears as a Z_2-gauge field over spacetime M^4. The assignment of the field with respect to the Z_2-gauge symmetry in [1] for the fermions is related to their chirality and for the gauge bosons and Higgs to the way they couple with fermions. However, since the standard model is not left–right symmetric, the field assignment with respect to the Z_2-gauge symmetry in [1] cannot be applied directly to the Weinberg–Salam model and the standard model for the electroweak–strong interaction. In [2, 3], we adopt a Z_2-gauge symmetry in CPT to reformulate the Weinberg–Salam model and the standard model for the electroweak–strong interaction. This also indicates that the entire CPT symmetry should be gauged.

In this letter, we take the discrete gauge symmetry to be what we call the restricted CPT symmetry, Θ, i.e. a symmetry with a CPT transformation as a single element, and show that the Higgs sector in the Weinberg–Salam model may be dealt with as an ansatz for the gauge fields with respect to this restricted CPT-gauge symmetry. In what follows, we first construct a general model with the gauge symmetry $G_L \times G_R \times \Theta$. We assign fermions, gauge bosons and Higgs into four sectors with respect to four elements of the group Θ. For the fermions, this assignment is directly related to their transformation property under Θ and

* Work supported in part by The National Natural Science Foundation of China.

for the gauge bosons and Higgs not only to their transformation property under Θ but also to their coupling to the fermions. As for the Lagrangian, we may obtain the conventional one by considering the gauge invariance. On the other hand, however, we may also get it in terms of the discrete symmetry and the Haar integral over it, as we have done in [1–3]. In order to apply it to the Weinberg–Salam model, what we need to do is just to take the field contents in the general model to be that of the Weinberg–Salam model. Finally, we end with some discussions and remarks.

Let us first construct a model of the $G_L \times G_R$ and the restricted CPT-gauge symmetry with leptons $\psi(x,h)$, $h \in \Theta$, Yang–Mills gauge potentials $A_\mu(x,h)$ and Higgs $\Phi(x,h)$. We assign them into four sectors according to four elements of the group Θ as follows:

$$
\begin{aligned}
&\psi(x,e) = -\psi(x,r^2) = \begin{pmatrix} L \\ R^r \end{pmatrix} \qquad \psi(x,r) = -\psi(x,r^3) = \begin{pmatrix} L^r \\ -R \end{pmatrix} \\
&A_\mu(x,e) = A_\mu(x,r^2) = \begin{pmatrix} L_\mu & 0 \\ 0 & R^r_\mu \end{pmatrix} \qquad A_\mu(x,r) = A_\mu(x,r^3) = \begin{pmatrix} L^r_\mu & 0 \\ 0 & R_\mu \end{pmatrix} \\
&\Phi_r(x,e) = \Phi_r(x,r^2) = \begin{pmatrix} \frac{\mu}{\lambda} & \phi \\ -\phi^{\dagger r} & \frac{\mu}{\lambda} \end{pmatrix} \qquad \Phi_r(x,r) = \Phi_r(x,r^3) = \begin{pmatrix} \frac{\mu}{\lambda} & \phi^r \\ -\phi^\dagger & \frac{\mu}{\lambda} \end{pmatrix} \\
&\Phi_{r^2}(x,e) = \Phi_{r^2}(x,r) = \Phi_{r^2}(x,r^2) = \Phi_{r^2}(x,r^3) = \begin{pmatrix} 2\frac{\mu}{\lambda} & 0 \\ 0 & 2\frac{\mu}{\lambda} \end{pmatrix} \\
&\Phi_{r^3}(x,e) = \Phi_{r^3}(x,r^2) = \begin{pmatrix} \frac{\mu}{\lambda} & -\phi \\ \phi^{\dagger r} & \frac{\mu}{\lambda} \end{pmatrix} \qquad \Phi_{r^3}(x,r) = \Phi_{r^3}(x,r^3) = \begin{pmatrix} \frac{\mu}{\lambda} & -\phi^4 \\ \phi^\dagger & \frac{\mu}{\lambda} \end{pmatrix}
\end{aligned}
\tag{1}
$$

where L and R are the left- and right-handed fermions, L_μ and R_μ the gauge potentials coupled to the fermions L and R and valued on the Lie algebras of the gauge group G_L and G_R which are supposed to be semi-simple respectively, L^r and R^r the CPT transformed fields of L and R etc, μ and λ two constants. From the assignments, it is easy to see that what is gauged is the group Θ with a special ansatz for the Higgs. Namely, the room for Higgs in such a model has not been fully used. On the other hand, however, it should be mentioned that the assignments (1) not only assign the fields to the elements of Θ but also imply that we arrange all fields into certain matrices. As was pointed out in [2,3], in fact, this arrangement is a working hypothesis for the sake of convenience in the forthcoming calculation. Of course, we must keep it in mind in order to avoid some extra constraints coming from this matrix arrangement.

From the general framework we have developed in [1,2], it follows the generalized connection one-form and the generalized curvature two-form:

$$
A(x,h) = A_\mu(x,h)\,\mathrm{d}x^\mu + \frac{\lambda}{\mu}\sum_g \Phi_g(x,h)\chi^g \qquad h \in \Theta \qquad g \in \Theta' = \Theta\backslash e \tag{2}
$$

$$
\begin{aligned}
F(h) &= \mathrm{d}A(h) + A(h) \otimes A(h) \\
&= \tfrac{1}{2}F_{\mu\nu}(h)\,\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu + \frac{\lambda}{\mu}\sum_g F_{\mu g}(h)\,\mathrm{d}x^\mu \otimes \chi^g \\
&\quad + \frac{\lambda^2}{\mu^2}\sum_{fg} F_{fg}(h)\chi^f \otimes \chi^g \qquad h \in \Theta \qquad f, g \in \Theta'.
\end{aligned}
\tag{3}
$$

Using the above assignment, we get the non-vanishing components at the element $e \in \Theta$ as follows:

$$F_{\mu\nu}(x,e) = \frac{1}{2}\begin{pmatrix} L_{\mu\nu} & 0 \\ 0 & R^r_{\mu\nu} \end{pmatrix}$$

$$F_{\mu r}(x,e) = -F_{\mu r^3}(x,e) = \frac{\lambda}{\mu}\begin{pmatrix} 0 & D_\mu\phi \\ -D_\mu\phi^{\dagger r} & 0 \end{pmatrix} \tag{4}$$

$$F_{rr^3}(x,e) = F_{r^3 r}(x,e) = -F_{rr}(x,e) = -F_{r^3r^3}(x,e) = \frac{\lambda^2}{\mu^2}\begin{pmatrix} \phi\phi^\dagger - \frac{\mu^2}{\lambda^2} & 0 \\ 0 & \phi^{\dagger r}\phi^r - \frac{\mu^2}{\lambda^2} \end{pmatrix}$$

where

$$H_{\mu\nu} = \partial_\mu H_\nu - \partial_\nu H_\mu - [H_\mu, H_\nu] \qquad H = L, R$$
$$D_\mu\phi = \partial_\mu\phi + L_\mu\phi - \phi R_\mu. \tag{5}$$

The non-vanishing components of the curvature at other elements can be given by means of the CPT transformations.

Having these building blocks, we may get the generalized gauge-invariant Lagrangian, including both the bosonic part and the fermionic part as well as their interactions via the generalized minimum coupling principle in the conventional way. From the field contents (1), it follows a Lagrangian of the ordinary type in gauge-invariant models. On the other hand, as in [1–3], we may also introduce the generalized gauge-invariant Lagrangian with respect to each element of Θ first, then take the Haar integral of them over Θ. Under these considerations we may get the same Lagrangian as usual, as was pointed out in [2, 3].

For the Lagrangian of the bosonic sector we have

$$\begin{aligned} \mathcal{L}_{\mathrm{YM-H}}(x,e) &= \mathcal{L}_{\mathrm{YM-H}}(x,r^2) \\ &= -\frac{1}{4N_L}\mathrm{Tr}_L(L_{\mu\nu}L^{\mu\nu}) - \frac{1}{4N_R}\mathrm{Tr}_R(R^r_{\mu\nu}R^{\mu\nu r}) \\ &\quad + 2\eta\frac{\lambda^2}{\mu^2}\{\mathrm{Tr}(D_\mu\phi(x)(D^\mu\phi(x))^\dagger) + (D_\mu\phi(x))^{\dagger r}D^\mu\phi(x)^r\} \\ &\quad - 4\eta^2\frac{\lambda^4}{\mu^4}\left\{\mathrm{Tr}\left(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2}\right)^2 + \mathrm{Tr}\left(\phi(x)^{\dagger r}\phi(x)^r - \frac{\mu^2}{\lambda^2}\right)^2\right\} \end{aligned} \tag{6}$$

$$\mathcal{L}_{\mathrm{YM-H}}(x,r) = \mathcal{L}_{\mathrm{YM-H}}(x,r^3) = \mathcal{L}^r_{\mathrm{YM-H}}(x,e)$$

where N_L and N_R are normalization constants, η is a metric parameter defined by $\eta = (\chi^g, \chi^{g^{-1}})$, $Dim(\eta) = [\mu^2]$. Here we suppose that both G_L and G_R are semi-simple. For the fermionic sector, the Lagrangian with respect to each element of Θ may also be given as follows:

$$\begin{aligned} \mathcal{L}_{\mathrm{F}}(x,e) &= \mathcal{L}_{\mathrm{F}}(x,r^2) \\ &= \mathrm{i}\bar{L}\gamma^\mu(\partial_\mu + L_\mu)L + \{\mathrm{i}\bar{R}\gamma^\mu(\partial_\mu + R_\mu)R\}^r - \lambda(\bar{L}\phi(x)R + \overline{R^r}\phi(x)^{\dagger r}L^r) \end{aligned} \tag{7}$$

$$\mathcal{L}_{\mathrm{F}}(x,r) = \mathcal{L}_{\mathrm{F}}(x,r^3) = \mathcal{L}^r_{\mathrm{F}}(x,e).$$

It is easy to get the entire Lagrangian for the model:

$$\mathcal{L}(x) = \mathcal{L}_{\mathrm{F}}(x) + \mathcal{L}_{\mathrm{YM-H}}(x) + \{CPT\} \tag{8}$$

where $\{CPT\}$ denotes the CPT-transformed version of the previous terms and

$$\begin{aligned}
\mathcal{L}_{\mathrm{F}}(x,e) &= \mathcal{L}_{\mathrm{F}}(x,r^2) \\
&= \mathrm{i}\bar{L}\gamma^{\mu}(\partial_\mu + L_\mu)L + \{\mathrm{i}\bar{R}\gamma^{\mu}(\partial_\mu + R_\mu)R\}^r - \lambda(\bar{L}\phi(x)R + \overline{R^r}\phi(x)^{\dagger^r}L^r) \\
\mathcal{L}_{\mathrm{YM-H}}(x) &= -\frac{1}{4N_L}\mathrm{Tr}_L(L_{\mu\nu}L^{\mu\nu}) - \frac{1}{4N_r}\mathrm{Tr}_R(R_{\mu\nu}R^{\mu\nu}) \\
&\quad + 2\eta\frac{\lambda^2}{\mu^2}\{\mathrm{Tr}(D_\mu\phi(x)(D^\mu\phi(x))^\dagger) + (D_\mu\phi(x))^\dagger D^\mu\phi(x)\} \\
&\quad - 4\eta^2\frac{\lambda^4}{\mu^4}\left\{\mathrm{Tr}\left(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2}\right)^2 + \left(\phi(x)^\dagger\phi(x) - \frac{\mu^2}{\lambda^2}\right)^2\right\}.
\end{aligned} \tag{9}$$

We are now ready to reformulate the Weinberg–Salam model. For simplicity, we deal with the case of only one family of leptons. It is straightforward to generalize to the case of three families. For the case at hand, we have

$$\begin{aligned}
&L(x) = \begin{pmatrix} \nu_l \\ l \end{pmatrix} \qquad R(x) = l_R \qquad \phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\
&L_\mu = -\mathrm{i}g\frac{\tau_i}{2}W^i_\mu + \mathrm{i}\frac{g'}{2}B_\mu \qquad R_\mu = \mathrm{i}g'B_\mu.
\end{aligned} \tag{10}$$

Thus

$$\begin{aligned}
L_{\mu\nu} &= -\mathrm{i}g\frac{\tau_i}{2}W^i_{\mu\nu} + \mathrm{i}\frac{g'}{2}B_{\mu\nu} \\
R_{\mu\nu} &= \mathrm{i}g'B_{\mu\nu} \\
D_\mu\phi &= \left(\partial_\mu - \mathrm{i}g\frac{\tau_i}{2}W^i_\mu - \mathrm{i}\frac{g'}{2}B_\mu\right)\phi.
\end{aligned} \tag{11}$$

Substituting these field contents into (6), (7) and taking the Haar integral, we get the Lagrangian in the Weinberg–Salam model plus its copy under CPT as follows:

$$\mathcal{L}(x) = \mathcal{L}_{\mathrm{F}}(x) + \mathcal{L}_{\mathrm{YM-H}}(x) + \{CPT\} \tag{12}$$

where that for the gauge bosons and Higgs is

$$\begin{aligned}
\mathcal{L}_{\mathrm{YM-H}}(x) &= -\frac{1}{4N_L}\frac{g^2}{2}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4N_Y}\frac{3g'^2}{2}B_{\mu\nu}B^{\mu\nu} + 4\eta\frac{\lambda^2}{\mu^2}\mathrm{Tr}(D_\mu\phi(x))(D^\mu\phi(x))^\dagger \\
&\quad - 8\eta^2\frac{\lambda^4}{\mu^4}\left\{\mathrm{Tr}\left(\phi(x)\phi(x)^\dagger - \frac{\mu^2}{\lambda^2}\right)^2\right\} + \text{const}
\end{aligned} \tag{13}$$

where N_L and N_Y are two normalization constants. Since $G_L = SU_L(2) \times U_Y(1)$ is not semi-simple, we should take a different normalization from that in (6). Also, the Lagrangian for leptons is

$$\begin{aligned}
\mathcal{L}_{\mathrm{F}}(x) &= \mathrm{i}\bar{L}(x)\gamma^\mu\left\{\partial_\mu - \left(\mathrm{i}g\frac{\tau_i W^i_\mu}{2} - \mathrm{i}g'\frac{B_\mu}{2}I_2\right)\right\}L + \mathrm{i}\bar{R}(x)\gamma^\mu(\partial_\mu + \mathrm{i}g'B_\mu)R \\
&\quad - \lambda(\bar{L}(x)\phi(x)R(x) + \bar{R}(x)\phi(x)^\dagger L(x)).
\end{aligned} \tag{14}$$

We may take the normalization of the coefficients of each term as follows:

$$N_L = \frac{g^2}{2} \qquad N_Y = \frac{3g'^2}{2} \qquad 4\eta\frac{\lambda^2}{\mu^2} = 1. \tag{15}$$

When $\mathrm{Tr}(\phi\phi^\dagger) = (\mu/\lambda)^2$, the Higgs potential takes its minimum value and the continuous gauge symmetry will spontaneously be broken down. Now we take the vacuum expectation value of ϕ to be

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad v = \sqrt{2}\frac{\mu}{\lambda} \tag{16}$$

and introduce a new field $\rho(x)$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+\rho}{\sqrt{2}} \end{pmatrix} \tag{17}$$

as well as the photon and Z boson via the Weinberg angle

$$\begin{aligned} A_\mu &= B_\mu \cos\theta_{\mathrm{W}} + W^3_\mu \sin\theta_{\mathrm{W}} \\ Z_\mu &= B_\mu \sin\theta_{\mathrm{W}} - W^3_\mu \cos\theta_{\mathrm{W}} \\ g\sin\theta_{\mathrm{W}} &= g'\cos\theta_{\mathrm{W}} = \frac{gg'}{\sqrt{g^2+g'^2}} = e. \end{aligned} \tag{18}$$

Using (15), we get

$$\sin^2\theta_{\mathrm{W}} = \frac{g'^2}{g^2+g'^2} \left\{= \frac{N_Y}{3N_L+N_Y}\right\}. \tag{19}$$

We also have

$$\begin{aligned} &\mathrm{Tr}\left\{D_\mu\phi(D_\mu\phi)^\dagger - \tfrac{1}{2}\left(\phi\phi^\dagger - \frac{\mu^2}{\lambda^2}\right)^2\right\} \\ &\qquad = \tfrac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{g^2}{4}(v+\rho)^2 W^-_\mu W^+_\mu + \tfrac{1}{8}(g^2+g'^2)(v+\rho)^2 Z_\mu Z_\mu \\ &\qquad\quad - \tfrac{1}{2}\rho^2\left(v^2 + v\rho + \frac{\rho^2}{4}\right) + \text{const.} \end{aligned} \tag{20}$$

It is easy to see that, as in the Weinberg–Salam model, A_μ and ν_l remain massless while fermion l together with gauge bosons $\mathrm{W}^\pm$ and Z become massive.

It is worth pointing out that in this letter we have adopted a renormalization in [2, 3] which is different from [1] so that, as in [2, 3], we get no constraints between the coupling constants and mass parameters other than the Higgs mass at the tree level:

$$M_{\mathrm{Higgs}} = v. \tag{21}$$

In fact, we might get a constraint for the Weinberg angle if we could take $N_L = N_Y$. As for the Higgs mass, it may also be released, as was pointed out in [2, 3].

One of the important points in this letter is the link between the restricted CPT-gauge symmetry and the Weinberg–Salam model. It is also straightforward to set up this link between the restricted CPT-gauge symmetry and the standard model as well. We will explore this link in detail elsewhere. Another important point is that the room for the Higgs has not been fully used. We may introduce more Higgs fields to enlarge the model so as to explain other physical phenomena related to the Higgs sector, such as CP violation [8, 9] and so on. On the other hand, what has been gauged in this letter is not the entire CPT symmetry. As was mentioned in [2, 3], the content and implication of the entire CPT symmetry is very rich. Therefore, gauging the CPT symmetry may shed light on some fundamental problems.

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